Detouring Skip Graph: Efficient Routing via Detour Routes on Skip Graph Topology

Takeshi Kaneko, Ryohei Banno, *Member, IEEE*, Kazuyuki Shudo, *Member, IEEE*, Kota Abe, *Member, IEEE*, and Yuuichi Teranishi, *Member, IEEE*

Skip graph is a distributed data structure that provides a scalable structured overlay network by routing in logarithmic time for resource location and dynamic node addition/deletion. However, most of the routing paths are quite longer than the shortest paths because each node in the network knows only its neighbors, rather than the global topology. In general, long routing paths lead to the high latency and the low fault tolerance. Herein, we propose Detouring Skip Graph, which performs more efficient routing through the use of detour routes. It does not require construction of extra links or modification of its topology; thereby, it shortens the paths without additional costs while maintaining the advantages of Skip Graph. Our evaluation experiments show that the proposed method tends to shorten the paths considerably, and in particular, that the average path length is approximately 20%–30% shorter than that of Skip Graph.

***Index Terms*—Detour route, routing algorithm, Skip Graph, structured overlay**

I. INTRODUCTION

An overlay network is an application-level logical network built on an existing network such as the Internet. Each node in overlay networks is organized in a decentralized manner and flexibly adapts to the dynamic underlying network [1]. Especially a structured overlay constructs an autonomous distributed network according to a specific data structure or some protocols; thereby providing reachability to target nodes, high scalability, high fault tolerance, and some useful functions. Owing to these properties, application to a variety of large-scale distributed systems has been proposed; e.g. distributed key/value stores [2], video streaming [3], and online games [4]. In recent years, application to the fields of IoT and Blockchain is also expected [5], [6].

Skip Graph [7] is a distributed data structure that provides a scalable structured overlay managing pairs of a key and a value. By hashing keys, it works as a distributed hash table (DHT) and archives good load balancing for data management. On the other hand, even if keys are not hashed, it constructs a balanced topology that performs routing in logarithmic time for resource location and dynamic node addition/deletion, unlike some typical DHTs [8], [9]. Moreover, Skip Graph without hashing supports range queries [10], [11] as a result of preserving the order of keys.

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T. Kaneko and K. Shudo are with Department of Mathematical and Computing Science, Tokyo Institute of Technology, Meguro-ku, Tokyo, Japan (e-mails: kaneko.t.ay@m.titech.ac.jp, shudo@is.titech.ac.jp).

R. Banno was with Department of Mathematical and Computing Science, Tokyo Institute of Technology, Meguro-ku, Tokyo, Japan. He is now with Department of Information and Communications Engineering, Kogakuin University, Hachioji-shi, Tokyo, Japan (e-mail: banno@computer.org).

K. Abe is with Graduate School of Engineering, Osaka City University, Osaka-shi, Osaka, Japan, and National Institute of Information and Communications Technology, Koganei-shi, Tokyo, Japan (e-mail: k-abe@osaka-cu.ac.jp).

Y. Teranishi is with National Institute of Information and Communications Technology, Koganei-shi, Tokyo, Japan, and Cybermedia Center, Osaka University, Ibaraki-shi, Osaka, Japan (e-mail: yuuichi.teranishi@ieee.org).

However, it is still a challenge for Skip Graph to ensure that each node takes full advantage of the existing links. The routing paths tend to be quite longer than the shortest paths because it knows only its neighbors, rather than the global topology. In general, an overlay network with long routing paths leads to the high latency and the low fault tolerance. In a homogeneous environment such as a local area network and a cloud network, a routing path length dominates the latency. In contrast, note that locality awareness [12], [13], [14], [15], which considers the proximity of the underlying network, is also important in a non-homogeneous environment on the Internet. Regarding fault tolerance, it is effective to use the stabilization methods [16], [17]. However, the probability of encountering a fault in a routing process increases as the path length increases.

Since most application mentioned above of overlay networks requires the high responsiveness and high reliability, shortening routing paths is a critical demand for overlays including Skip Graph. We propose an extension of Skip Graph which is called Detouring Skip Graph. It shortens the path lengths through a more efficient use of the existing links while maintaining the advantages of Skip Graph. Specifically, its routing algorithm is different from that of Skip Graph in two ways: each node 1) utilizes detour routes and 2) traverses adjacent nodes from its maximum level.

This paper is an extended version of our previous work [18]. The main differences are comprehensive comparison experiments and mathematical analysis. This paper provides comparisons in maximum path lengths and path length distribution, an additional comparison target `searchNLIOp`, and additional evaluation scenarios (in Section IV); and proves correctness and complexity of the proposed method (in Section III-E and III-F). This paper also provides all the pseudocodes including `searchDSGOp` (Algorithm 3). The rest of this paper is organized as follows. Section II presents an overview of Skip Graph and the related work on shortening path lengths. Section III presents Detouring Skip Graph in detail. Section IV presents the evaluation experiments for the proposed method and the results. Finally, Section V presents the conclusion of

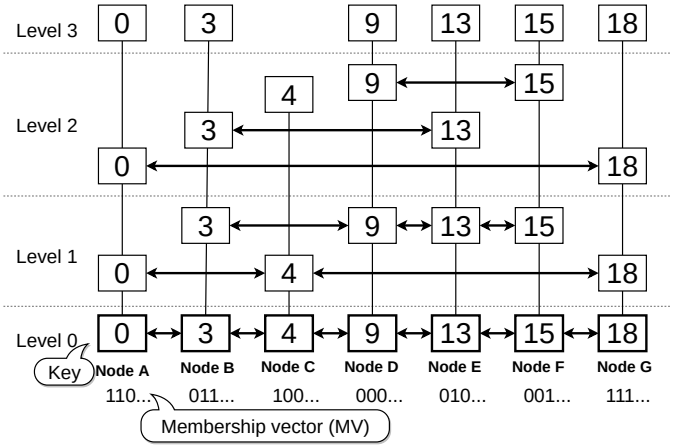

Fig. 1. An example of Skip Graph.

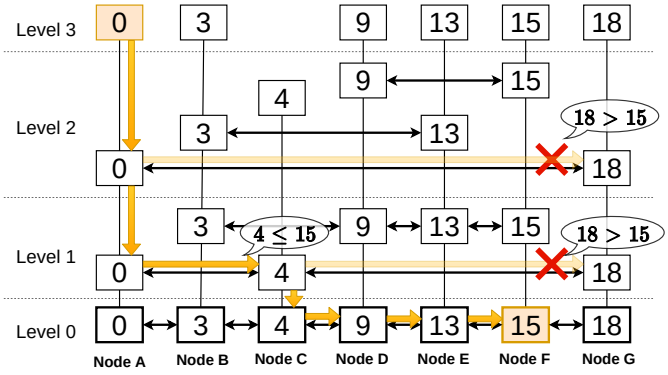

Fig. 2. A `searchOp` routing to search a node whose key is 15 from a node A.

this study.

II. Related Work

A. Skip Graph

Skip Graph is a distributed data structure designed based on Skip List [19], and each node belongs to multiple sorted doubly linked lists. Fig. 1 shows an example of a topology of Skip Graph. Each node has a key in a totally ordered set and a random string called membership vector (MV), which plays a key role in constructing the topology of Skip Graph. In the figure, the alphabet that are elements of MV is $\{0, 1\}$. In a linked list at level l, the leading l digits of the MV of every node is the same as that of all others. Particularly at level 0, all nodes belong to one linked list. Therefore, each node belongs to $O(\log n)$ linked lists. Further, by using the same method as Skip List, Skip Graph achieves a routing path length of $O(\log n)$ for a query to search a key k_{target}. A detailed explanation of the routing algorithm is presented below, where it is assumed that the keys in the linked lists are sorted in ascending order from left to right.

Algorithm 4 is a pseudocode of routing algorithm for search queries of Skip Graph. Suppose a node $v_{current}$ in Skip Graph is receiving a query `searchOp` to search a node that has a specific key. The query has three information $(v_{start}, k_{target}, l_{prev})$: a start node v_{start}, a target key k_{target}, and the level l_{prev} at which the previous node sends the query. If $v_{current}.key$, the key of $v_{current}$, is equal to k_{target}, $v_{current}$ sends a query `foundOp` to v_{start} since it means that $v_{current}$ is the target node. If $v_{current}.key < k_{target}$, $v_{current}$ traverses the right adjacent nodes at the levels from l_{prev} in descending order and sends a search query `searchOp` to the first adjacent node v_{next} where $v_{next}.key \leq k_{target}$. If $v_{current}.key > k_{target}$, $v_{current}$ traverses the left adjacent nodes and sends a search query `searchOp` to the next node in a similar manner. If $v_{current}.key \neq k_{target}$ and $v_{current}$ cannot find such a next node, $v_{current}$ sends a query `notFoundOp` to v_{start} since it means that there are no target nodes in the topology.

Fig. 2 shows a routing process when a query to search a node whose key is $k_{target} = 15$ is issued at a node A on the condition that each node follows the above method in the topology shown in Fig. 1. Then, the key sequence of the nodes on the routing path is $(0, 4, 9, 13, 15)$, and the path length is 4. This routing specifies the target node is a node F.

B. Shortening Path Lengths

The path length is 4 in Fig. 2, however, there are shorter paths in reality. For instance, if the node A chooses a node G at level 2 instead of a node C at level 1 as the next node, the key sequence of the nodes on the path would be $(0, 18, 15)$, and the path length would be 2 (which is shorter than 4). Thus, the routing of Skip Graph is inefficient in that it cannot fully utilize the existing links. Therefore, various approaches have been proposed for shortening the path lengths.

In the above example, at the route from the node A to the node G, the magnitude relationship between the key of the current node and the target key k_{target} is reversed. Routes like this are hereinafter referred to as "detour routes." The proposed method (described later in detail) utilizes detour routes. It is similar to the method proposed by Higuchi et al. [20] in terms of use of detour routes. However, the subject of their method is not ordinary Skip Graph but Skip Graph whose topology is balanced by means of using linear hashing preserving the order.

There are already several methods to shorten path lengths of Skip Graph routing: e.g., methods that involve construction of extra links in the original Skip Graph and appropriate routing on the topology including the links [21], [22], and methods that reconstruct or refine the unbalanced topology resulting from randomly generated MVs into the ideal topology [23], [24], [25], [26]. However, the construction of extra links and the modification of the topology lead to an increase in necessary transfer messages and maintenance costs. Our proposed method has an advantage in that it shortens the path lengths without such additional costs.

III. Proposed Method

The main idea of Detouring Skip Graph is the utilization of detour routes. Moreover, the idea can be combined with the technique of traversing from maximum levels. This section presents these two techniques and the details of Detouring Skip Graph whose routing algorithm combines them.

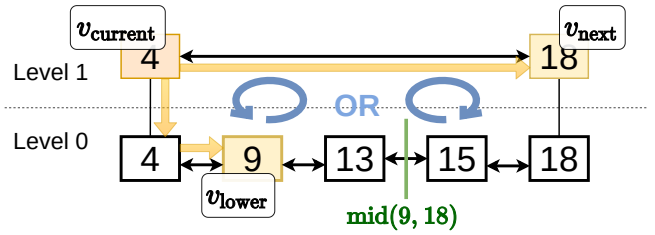

Fig. 3. Should the node $v_{current}$ select the detour route?

In the following, let K be a set of keys. To simplify, we assume that K is a subset of real numbers[1]. Thus, arithmetic operations and absolute values are well-defined on K. Moreover, we also assume that there is at most one node for each key. In practice, the uniqueness of keys is not always guaranteed because replication [28], [29] is often used to provide data availability and fault tolerance. A simple way to eliminate duplicate keys is to add some random bits on the right side of identical keys [30].

A. Utilizing Detour Routes

The routing algorithm of Detouring Skip Graph utilizes detour routes. The idea is based on the following argument.

Fig. 3 shows a part of the topology of Fig. 1. Let v_{next} and v_{lower} be the right neighbors of a node $v_{current}$ at level 1 and 0, respectively. Now, suppose $v_{current}$ is receiving a query to search a node whose key equals $k_{target} \in K$ where $9 \leq k_{target} < 18$. In a situation where $v_{current}$ follows the routing algorithm of Skip Graph, it selects v_{lower} as the next node since $v_{next}.key > k_{target}$ and $v_{lower}.key \leq k_{target}$. If $k_{target} = 15$, the key sequence of the nodes on the path would be $(4, 9, 13, 15)$, and the path length would be 3. However, in a situation where it selects v_{next} as the next node, i.e., it uses the detour route, the key sequence would be $(4, 18, 15)$, and the path length would be 2. The path length of the latter is shorter than that of the former. The effectiveness of such detour routes depends on the position of the target key; if $k_{target} = 13$, the path length of using the detour route would be longer than that of using the ordinary route.

As shown in Fig. 3, it can be determined from the center of the node sequence at the lower level (level 0 in Fig. 3) whether $v_{current}$ should select v_{next} or v_{lower} to shorten the path length. This section presents the routing algorithm that each node determines the next node based on a detour criterion at each level.

Algorithm 1 is a pseudocode of this algorithm. Herein, $v_{current}$ traverses the adjacent nodes in the same way as Skip Graph when a node $v_{current}$ receives a query `searchDROp`. However, if $v_{current}$ judges that using a detour route is better than not using it, $v_{current}$ selects the end node of the detour route as the next node. Function $closeToRight(k_{target}, k_{left}, k_{right})$ can be used to make this judgment. Let I_{right} be $\{k \in K \mid k \geq k_{right}\}$. The function

[1]As a mathematical fact, any countable totally ordered set can be order embedded into the set of rational numbers [27], which is a subset of real numbers.

Algorithm 1: `searchDROp` in node $v_{current}$

```
/* utilizing Detour Routes                                          */
upon receiving ⟨searchDROp, v_start, k_target, l_prev⟩ then
    if v_current.key = k_target then
        send ⟨foundOp, v_current⟩ to v_start ;
        return;
    else if v_current.key < k_target then
        for l_current ← l_prev downTo 0 do
            v_next ← v_current.neighbors[R][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≤ k_target then
                send ⟨searchDROp, v_start, k_target, l_current⟩ to
                    v_next ;
                return;
            else if l_current > 0 then
                v_lower ← v_current.neighbors[R][l_current − 1];
                if closeToRight(k_target, v_lower.key, v_next.key)
                    then
                    send ⟨searchDROp, v_start, k_target, l_current⟩ to
                        v_next ;
                    return;
    else
        for l_current ← l_prev downTo 0 do
            v_next ← v_current.neighbors[L][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≥ k_target then
                send ⟨searchDROp, v_start, k_target, l_current⟩ to
                    v_next ;
                return;
            else if l_current > 0 then
                v_lower ← v_current.neighbors[L][l_current − 1];
                if ¬ closeToRight(k_target, v_next.key, v_lower.key)
                    then
                    send ⟨searchDROp, v_start, k_target, l_current⟩ to
                        v_next ;
                    return;
        send ⟨notFoundOp, v_current⟩ to v_start;
function closeToRight(k_target, k_left, k_right)
    k_mid ← mid(k_left, k_right);
    return k_mid < k_target;
```

returns *true* if the signed distance from k_{target} to I_{right} is smaller than that from $mid(k_{left}, k_{right})$ to I_{right}, i.e., $k_{right} - k_{target} < k_{right} - mid(k_{left}, k_{right})$; otherwise, it returns *false*. Intuitively, it means that k_{target} is closer to k_{right} than $mid(k_{left}, k_{right})$. Further, mid is a design parameter defined as a function before the construction of the topology to satisfy that the following (1) and (2) are approximately equal for the set V of all participating nodes at any point in time and any $k_1, k_2 \in K$ $(k_1 \leq k_2)$;

$$\#\left\{ k \in K \;\middle|\; k_1 \leq k \leq k_{mid} \wedge^{\exists} v \in V, v.key = k \right\} \quad (1)$$

$$\#\left\{ k \in K \;\middle|\; k_{mid} \leq k \leq k_2 \wedge^{\exists} v \in V, v.key = k \right\} \quad (2)$$

where $k_{mid} = mid(k_1, k_2)$. (1) means the number of nodes v where $v.key \in [k_1, k_{mid}]$, and (2) means the number of nodes v where $v.key \in [k_{mid}, k_2]$. Thus, $mid(k_1, k_2)$ implies the center between k_1 and k_2 for the key distribution of the

participating nodes. We give some examples of defining mid. Let $v.key$, the key of a node v, be regarded as a random variable.

If

$$K = \{0, 1, \ldots, n-1\} \text{ and } P\{v.key = k\} = \frac{1}{n}, \quad (3)$$

then

$$mid(k_1, k_2) := \frac{k_1 + k_2}{2}. \quad (4)$$

In Fig. 3 and for this mid definition, $mid(9, 18) = \frac{27}{2}$ holds, and thus $closeToRight(k_{target}, 9, 18) \iff \frac{27}{2} < k_{target}$. This means the target node is located to the right of the estimated center if $\frac{27}{2} < k_{target}$, and is located to the left if $k_{target} \leq \frac{27}{2}$.

Next, if

$$K = [\alpha, \beta] \text{ and } P\{v.key \leq k\} = \int_{\alpha}^{k} c\kappa^{\gamma} d\kappa \quad (5)$$

with constants $\alpha, \beta, \gamma, c \in \mathbb{R}$ where $\alpha < \beta, \gamma > 1$ and $\int_{\alpha}^{\beta} ck^{\gamma} dk = 1$; then

$$\begin{aligned} \mathbb{E}\left[\frac{(1)}{|V|}\right] &= P\{k_1 \leq v.key \leq k_{mid}\} \\ &= c\frac{k_{mid}^{\gamma+1} - k_1^{\gamma+1}}{\gamma+1} \end{aligned} \quad (6)$$

and

$$\begin{aligned} \mathbb{E}\left[\frac{(2)}{|V|}\right] &= P\{k_{mid} \leq v.key \leq k_2\} \\ &= c\frac{k_2^{\gamma+1} - k_{mid}^{\gamma+1}}{\gamma+1}. \end{aligned} \quad (7)$$

When (1) and (2) are approximately equal, i.e.:

$$c\frac{k_{mid}^{\gamma+1} - k_1^{\gamma+1}}{\gamma+1} = c\frac{k_2^{\gamma+1} - k_{mid}^{\gamma+1}}{\gamma+1}, \quad (8)$$

we get

$$k_{mid} = \left(\frac{k_1^{\gamma+1} + k_2^{\gamma+1}}{2}\right)^{\frac{1}{\gamma+1}}. \quad (9)$$

Thus, in this case, we should define (9) as mid.

By defining mid in this way, $mid(v_{lower}.key, v_{next}.key)$ or $mid(v_{next}.key, v_{prev}.key)$ refers to a key estimation of the center of the lower-level node sequence. Thus, $closeToRight$ plays the appropriate role of a detour judgment.

In practice, it is difficult to obtain the distribution of the keys beforehand. However, from the evaluation experiments presented in Section IV, we observed that it is effective in many cases for shortening path lengths by defining mid as:

$$mid(k_1, k_2) := \frac{k_1 + k_2}{2}. \quad (10)$$

Fig. 4 shows a routing process when a query to search a node whose key is $k_{target} = 15$ is issued at a node A on the condition that each node follows Algorithm 1 in the topology shown in Fig. 1. The key sequence of the nodes on the path is $(0, 18, 15)$, and the path length is 2, which is shorter than that of Fig. 2.

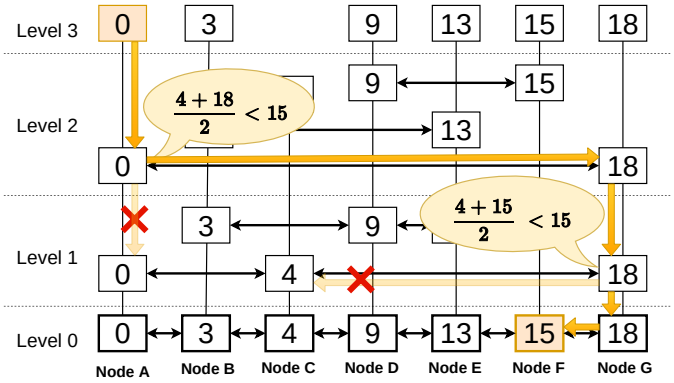

Fig. 4. A `searchDROp` routing to search a node whose key is 15 from a node A where $mid(k_1, k_2) = \frac{k_1+k_2}{2}$.

B. Traversing from the Maximum Level

In addition to utilizing detour routes, we can improve the routing of Skip Graph by making each node to traverse from the maximum level.

As discussed in Section II-A, in the routing of Skip Graph, a node $v_{current}$ traverses the adjacent nodes at the levels from the reception level l_{prev} in descending order and determines the first adjacent node v_{next} that satisfies the condition as the next node. The levels are monotonically decreasing for the entire routing. However, there are cases where an adjacent node at a level larger than l_{prev} satisfies the condition. Moreover, the larger the level at which $v_{current}$ sends a query to the next node, the larger the difference in the keys between adjacent nodes. Therefore, the difference between the key of the next node and the target key k_{target} when traversing from level $v_{current}.maxLevel$ is smaller than or equal to the difference when traversing from level l_{prev}, where $v.maxLevel$ is the maximum level of a node v. Thus, it is effective in shortening the path lengths that each node traverses from its maximum level.

Algorithm 2 is a pseudocode of this algorithm. When a node $v_{current}$ receives a query `searchMLOp`, $v_{current}$ traverses the adjacent nodes from $v_{current}.maxLevel$ and sends a query `searchMLOp` to the first node that satisfies the condition. This routing differs from that of Skip Graph only in the start level of traversing. Note that it is possible to use binary search for finding a next node instead of linear search, which is faster, but this code uses the latter for simplicity.

Fig. 5 shows a routing process when a query to search a node whose key is $k_{target} = 15$ is issued at a node A on the condition that each node follows Algorithm 2 in the topology shown in Fig. 1. The key sequence of the nodes on the path is $(0, 4, 9, 15)$, and the path length is 3, which is shorter than that of Fig. 2. While `searchMLOp` uses a vertical link from each level toward the maximum level at each node, the routing algorithm `searchOp` of Skip Graph does not use it. However, the change does not affect the topology because vertical links are virtual links involving only the software process of the node. Thus, `searchMLOp` requires no modification of the topology.

It should be noted that Algorithm 2 has the disadvantage of

Algorithm 2: `searchMLOp` in node $v_{current}$

```
/* traversing from Max Level                                   */
upon receiving ⟨searchMLOp, v_start, k_target⟩ then
    if v_current.key = k_target then
        send ⟨foundOp, v_current⟩ to v_start ;
        return;
    else if v_current.key < k_target then
        for l_current ← v_current.maxLevel downTo 0 do
            v_next ← v_current.neighbors[R][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≤ k_target then
                send ⟨searchMLOp, v_start, k_target⟩ to v_next;
                return;
    else
        for l_current ← v_current.maxLevel downTo 0 do
            v_next ← v_current.neighbors[L][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≥ k_target then
                send ⟨searchMLOp, v_start, k_target⟩ to v_next;
                return;
    send ⟨notFoundOp, v_current⟩ to v_start
```

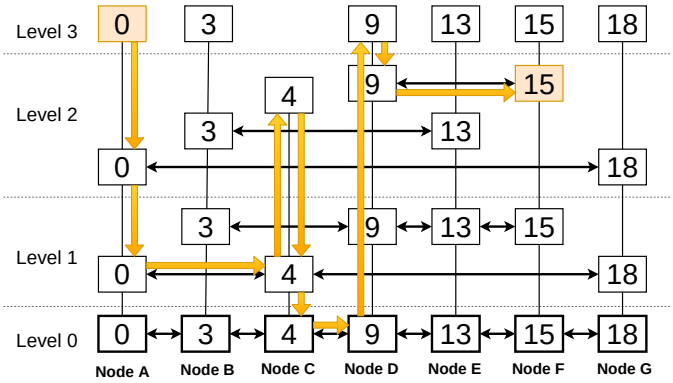

Fig. 5. A `searchMLOp` routing to search a node whose key is 15 from a node A.

increasing the computation costs incurred between receiving a query and determining the next node, although it has the advantage of shortening the path lengths. Let l_{MAX} be the maximum value of the maximum levels of all nodes, which is $O(\log n)$, and let H be the path length, which is $O(\log n)$. Then, the sum of the time required for each node on a routing path to determine the next node until finishing a routing process, except for the communication time and the I/O processing time, is $O(l_{MAX}+H) = O(\log n)$ for Skip Graph. On the other hand, in the case of the following Algorithm 2, it is $O(H \cdot l_{MAX}) = O(\log^2 n)$. Especially when using binary search, it is $O(H \log l_{MAX}) = O(\log n \cdot \log(\log n))$. These are inferior to Skip Graph in terms of computational complexity. However, the time taken for a routing process is typically dominated by the communication time, hence it is more important to shorten path lengths in most cases.

Algorithm 3: `searchDSGOp` in node $v_{current}$

```
/* Detouring Skip Graph                                        */
upon receiving ⟨searchDSGOp, v_start, k_target⟩ then
    if v_current.key = k_target then
        send ⟨foundOp, v_current⟩ to v_start ;
        return;
    else if v_current.key < k_target then
        for l_current ← v_current.maxLevel downTo 0 do
            v_next ← v_current.neighbors[R][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≤ k_target then
                send ⟨searchDSGOp, v_start, k_target⟩ to v_next ;
                return;
            else if l_current > 0 then
                v_lower ← v_current.neighbors[R][l_current − 1];
                if closeToRight(k_target, v_lower.key, v_next.key)
                  then
                    send ⟨searchDSGOp, v_start, k_target⟩ to v_next ;
                    return;
    else
        for l_current ← v_current.maxLevel downTo 0 do
            v_next ← v_current.neighbors[L][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≥ k_target then
                send ⟨searchDSGOp, v_start, k_target⟩ to v_next ;
                return;
            else if l_current > 0 then
                v_lower ← v_current.neighbors[L][l_current − 1];
                if ¬ closeToRight(k_target, v_next.key, v_lower.key)
                  then
                    send ⟨searchDSGOp, v_start, k_target⟩ to v_next ;
                    return;
    send ⟨notFoundOp, v_current⟩ to v_start;
```

C. Detouring Skip Graph: Combining Two Improvements

Because the two improved routing algorithms described above are independent changes from the routing algorithm of Skip Graph, an algorithm combining them can be defined naturally. We refer an extension of Skip Graph that performs such routing as Detouring Skip Graph.

Algorithm 3 is a pseudocode of this routing algorithm. When node $v_{current}$ receives a query `searchDSGOp`, $v_{current}$ traverses the adjacent nodes from $v_{current}.maxLevel$ in the same way as described in Section III-B and sends a query `searchDSGOp` to the first node that satisfies the condition. In the process, `searchDSGOp` uses detour routes based on the detour judgment as described in Section III-A.

Fig. 6 shows a routing process when a query to search a node whose key is $k_{target} = 12$ is issued at a node A on the condition that each node follows Algorithm 3 in the topology. The key sequence of the nodes on the path is $(0, 18, 15, 12)$, and the path length is 3.

From the foregoing, Detouring Skip Graph can bring about the shortening of the path lengths for search queries. Unlike the existing methods discussed in Section II-B, it does not require construction of extra links or modification of its

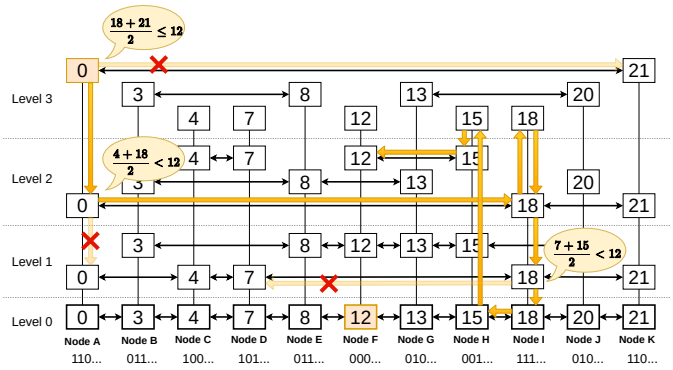

Fig. 6. A `searchDSGOp` routing to search a node whose key is 12 from a node A where $mid(k_1, k_2) = \frac{k_1+k_2}{2}$.

topology. Therefore, there is no increase in message transfer and management costs, and it maintains the good properties of Skip Graph. Additionally, the extension is simple and can be easily applied to existing Skip Graph.

D. Reachability of Detouring Skip Graph

Detouring Skip Graph has a property that the key sequence of the nodes on a routing path is not necessarily monotonic in order since the path is via detour routes, while Skip Graph does not have the property. You might consider that the routing process may lead to an infinite loop. However, the reachability is guaranteed, which is proved in this section.

Suppose a search query whose target key is k is being issued. Then, we introduce the following notations:

- Let $(v_1, v_2, \ldots)$ be the node sequence on the routing path.
- $S_k \coloneqq \{\, x \in K \mid x < k \,\}$.
- $T_k \coloneqq \{\, x \in K \mid x > k \,\}$.
- Let $(v_{s_1}, v_{s_2}, \ldots)$ be the subsequence of $(v_i)_i$, whose elements are all v_i satisfying $v_i.key \in S_k$.
- Let $(v_{t_1}, v_{t_2}, \ldots)$ be the subsequence of $(v_i)_i$, whose elements are all v_i satisfying $v_i.key \in T_k$.
- Let binary relation $\prec_k$, $\succ_k$ on $S_k \cup \{k\} \times T_k \cup \{k\}$ be:

$$\prec_k \coloneqq \left\{ \begin{array}{l} (x,y) \in \\ S_k \cup \{k\} \times T_k \cup \{k\} \end{array} \middle| \; closeToRight(k,x,y) \right\} \tag{11}$$

$$\succ_k \coloneqq \left\{ \begin{array}{l} (x,y) \in \\ S_k \cup \{k\} \times T_k \cup \{k\} \end{array} \middle| \; \neg closeToRight(k,x,y) \right\}. \tag{12}$$

Intuitively, $x \prec_k y$ implies that y is closer to k than x, and $x \succ_k y$ implies that x is closer to k than y.

The query reaches the target node if and only if $(v_i)_i$ is a finite sequence. Thus, it is sufficient to show that $(v_i)_i$ is finite. If both $(v_{s_i}.key)_i$ and $(v_{t_i}.key)_i$ are strictly approaching k (i.e., they are strictly increasing and strictly decreasing, respectively), $(v_i.key)_i$ converges to the key of the target node in finite steps and $(v_i)_i$ is a finite sequence.

Next, to show the strict monotonicity of each subsequence, the function mid used as the detour judgment must exhibit the following properties.

Property 1:

$$^{\forall}x, y \in K, x \le y \Rightarrow x \le mid(x,y) \le y \tag{13}$$

Property 2: $^{\forall}x, x', y, y' \in K$,

$$\max(x,x') \le y \Rightarrow [x \le x' \Leftrightarrow mid(x,y) \le mid(x',y)] \tag{14}$$

$$x \le \min(y,y') \Rightarrow [y \le y' \Leftrightarrow mid(x,y) \le mid(x,y')] \tag{15}$$

These properties are satisfied whenever mid is defined as the median estimation based on any probability distribution, e.g., $mid(x,y) \coloneqq \frac{x+y}{2}$. Then, the following lemmas hold.

Lemma 1: $^{\forall}x, x' \in S_k \cup \{k\}, ^{\forall}y, y' \in T_k \cup \{k\}$,

$$x \prec_k y \Rightarrow x' \le x \Rightarrow x' \prec_k y \tag{16}$$

$$x \prec_k y \Rightarrow y' \le y \Rightarrow x \prec_k y' \tag{17}$$

$$x \succ_k y \Rightarrow x \le x' \Rightarrow x' \succ_k y \tag{18}$$

$$x \succ_k y \Rightarrow y \le y' \Rightarrow x \succ_k y' \tag{19}$$

Proof: Suppose $x \prec_k y$ and $x' \le x$. From $\max(x,x') \le k \le y$ and Property 2, we have $mid(x',y) \le mid(x,y)$. Since $x \prec_k y$ means $mid(x,y) < k$, $mid(x',y) < k$ ($\Leftrightarrow x' \prec_k y$) holds. The other propositions can also be shown in the same way. ■

Lemma 2: $^{\forall}x_1, x_2 \in S_k \cup \{k\}, ^{\forall}y_1, y_2 \in T_k \cup \{k\}$,

$$\left[^{\exists}x \in S_k \cup \{k\} \text{ s.t. } (x \prec_k y_1 \wedge x \succ_k y_2)\right] \Rightarrow y_1 < y_2 \tag{20}$$

$$\left[^{\exists}y \in T_k \cup \{k\} \text{ s.t. } (x_1 \succ_k y \wedge x_2 \prec_k y)\right] \Rightarrow x_1 > x_2 \tag{21}$$

Proof: Suppose there exists $x \in S_k$ such that $x \prec_k y_1$ and $x \succ_k y_2$. If $y_1 \ge y_2$, then $x \succ_k y_1$ from $x \succ_k y_2$ and Lemma 1, however it contradicts $x \prec_k y_1$. Therefore, we have that $y_1 < y_2$. The latter proposition can also be established in the same way. ■

These lemmas derive the following theorem.

Theorem 1: $(v_{s_i}.key)_i$ and $(v_{t_i}.key)_i$ are strictly increasing and strictly decreasing, respectively.

Proof: It is sufficient for each step i to show that:

$$\begin{cases} j > 1 \Rightarrow v_{s_j}.key > v_{s_{j-1}}.key & (\text{ if } ^{\exists}j \text{ s.t. } s_j = i) \\ j > 1 \Rightarrow v_{t_j}.key < v_{t_{j-1}}.key & (\text{ if } ^{\exists}j \text{ s.t. } t_j = i), \end{cases} \tag{22}$$

where step i represents the process on the i-th node in the routing path. This can be shown using mathematical induction.

Suppose (22) holds at step $1, 2, \ldots, i$.

1) If $v_i.key = k$, then step $i+1$ does not exist because the routing process is complete (and the current node sends a query `foundOp` to the start node).
2) If $v_i.key \in S_k$, then the four cases are considered: (i) $v_{i+1}.key \in S_k$, (ii) $v_{i+1}.key \in T_k$, (iii) $v_{i+1}.key = k$, and (iv) $v_{i+1}.key$ does not exist. In case (i) and (iii), (22) holds at step $i+1$ because $v_i.key < v_{i+1}.key$ and $v_{i+1}.key \notin S_k \cup T_k$, respectively. In case (iv), step $i+1$ does not exist because the routing process is complete (and the current node sends a query `notFoundOp` to the start node). In case (ii), because a detour route is used, exist j and $x \in S_k \cup \{k\}$ such that $t_j = i+1$, $v_i.key < x$, and $x \prec_k v_{t_j}.key$. If $j > 1$, then it implies that a detour route was used at step t_{j-1} and that no detour route was used at step $t_{j-1}+1, t_{j-1}+2, \ldots, i-1$

owing to the definition of subsequence $(v_{t_{j'}})_{j'}$. Thus, $v_{t_{j-1}+1}.key, v_{t_{j-1}+2}.key, \ldots, v_i.key \in S_k$, and there exists $y \in T_k \cup \{k\}$ such that $y < v_{t_{j-1}}.key$ and $v_{t_{j-1}+1}.key \succ_k y$. From Lemma 1, we have $v_{t_{j-1}+1}.key \succ_k v_{t_{j-1}}.key$. In addition, $v_{t_{j-1}+1}.key < v_{t_{j-1}+2}.key < \cdots < v_i.key$ holds by the induction hypothesis. From Lemma 1, we have $v_i.key \succ_k v_{t_{j-1}}.key$. Thus, $v_{t_j}.key < v_{t_{j-1}}.key$ holds because of Lemma 2, i.e., (22) holds at step $i+1$.

3) If $v_i.key \in T_k$, then (22) holds at step $i+1$, which can be shown in the same way as 2).

■

Therefore, the reachability for any search query is guaranteed, regardless of whether there is a node with a target key k in the topology.

E. Correctness of Detouring Skip Graph

In this section, we prove the correctness of any search query of Detouring Skip Graph. Thus, the goal of this section is to ensure that a search operation returns a query `foundOp` if the target node v exists in the topology and that it returns a query `notFoundOp` if the target node v does not exist.

From III-D, the node sequence $(v_i)_i$ on a routing path is a finite sequence. Let the length of the sequence be M, i.e., v_M denotes the last node. Moreover, Algorithm 3 indicates that v_M sends `foundOp` if $v_M.key = k$ and that v_M sends `notFoundOp` if $v_M.key \neq k$. Thus, the correctness is guaranteed if the following theorem holds.

Theorem 2:

$$v_M.key = k \Rightarrow^{\exists} v \in V, v.key = k \tag{23}$$

$$v_M.key \neq k \Rightarrow^{\forall} v \in V, v.key \neq k \tag{24}$$

where V denotes the participating nodes (when v_M is in the routing process).

Proof: (23) holds since $v_M \in V$. Suppose $v_M.key \neq k$. It means that v_M cannot find a next node from the adjacent nodes of v_M.

1) If $v_M.key \in S_k$:
   a) If there exists the right adjacent node of v_M at level 0, let u be the node. Then, $u.key \in T_k$ since v_M cannot find a next node. There is only one sorted linked list at level 0, and the set of all the nodes equals V. It means that there does not exist $v \in V$ such that $v_M.key < v.key < u.key$. Thus, $^{\forall}v \in V, v.key \in S_k \cup T_k$, thereby $^{\forall}v \in V, v.key \neq k$ holds.
   b) Otherwise, v_M has no right adjacent node. It means that $v_M.key$ is the largest key in the topology. Thus, $^{\forall}v \in V, v.key \in S_k$, thereby $^{\forall}v \in V, v.key \neq k$ holds.
2) If $v_M.key \in T_k$, $^{\forall}v \in V, v.key \neq k$ holds, which can be shown in the same way as 1).

Therefore, (24) holds. ■

F. Complexity of Detouring Skip Graph

The search query in Skip Graph with n nodes takes expected $O(\log n)$ messages [7]. In this section, we prove that the message complexity for a search query of Detouring Skip Graph is also expected $O(\log n)$.

We introduce the following notations for Detouring Skip Graph with alphabets Σ.

- Let $m(v) \in \Sigma^{\infty}$ be the membership vector of a node $v \in V$.
- Let $|w|$ be the length of a word w, with $|w| = \infty$ if $w \in \Sigma^{\infty}$.
- Write $w \uparrow i$ for the prefix of a word w of length i.
- Write $w_1 \wedge w_2$ for the longest common prefix of words w_1 and w_2.

Suppose a search query whose target key is k is being issued. Without loss of generality, we assume that $v_1.key \in S_k$. Then, the goal of this section is to ensure that the number M of nodes in the routing path is expected $O(\log n)$ since the query takes $O(M)$ messages.

Lemma 3: $^{\forall}i, j$,

$$s_i < t_j \Rightarrow v_{s_i}.key \prec_k v_{t_j}.key \tag{25}$$

$$s_i > t_j \Rightarrow v_{s_i}.key \succ_k v_{t_j}.key \tag{26}$$

Proof: These propositions hold from Lemma 1 and Theorem 1. ■

Lemma 4: A sequence $(|m(v_1) \wedge m(v_i)|)_{i=1}^{M}$ is weakly decreasing.

Proof: Suppose, for proof by contradiction, that

$$\begin{aligned} &^{\exists}i \in \{1, \ldots, M-1\} \\ &\text{s.t. } |m(v_1) \wedge m(v_{i+1})| > |m(v_1) \wedge m(v_i)|. \end{aligned} \tag{27}$$

Let l be $|m(v_1) \wedge m(v_i)|$ and let j be the maximum number of $j' \in \{1, \ldots, i-1\}$ satisfying $|m(v_1) \wedge m(v_{j'})| \geq l+1$ (there exists such a j since $|m(v_1) \wedge m(v_1)| = \infty \geq l+1$).

1) If $v_j.key \in S_k$, then from $|m(v_1) \wedge m(v_j)| \geq l+1$ and $|m(v_1) \wedge m(v_{j+1})| \leq l$, we have $l_1 \leq l$ where $l_1 = |m(v_j) \wedge m(v_{j+1})|$. It means that the node v_{j+1} is the right adjacent node of v_j at level $l_1 (\leq l)$. In addition, from $|m(v_1) \wedge m(v_j)| \geq l+1$ and $|m(v_1) \wedge m(v_{i+1})| \geq l+1$, we have $l_2 \geq l+1$ where $l_2 = |m(v_j) \wedge m(v_{i+1})|$. It means that there exists a right adjacent node u of v_j at level $l_2 (\geq l+1)$ such that $v_{j+1}.key < k < u.key \leq v_{i+1}.key$ (possibly $u = v_{i+1}$). From Lemma 3 and Lemma 1, $v_{j+1}.key \prec_k u.key$ holds. Let u' be the right adjacent node of v_j at level $l_1 + 1$, and $k < u'.key \leq u.key$. From Lemma 1, $v_{j+1}.key \prec_k u'.key$ holds. On the other hand, since v_j selects v_{j+1} as the next node, $v_{j+1}.key \succ_k u'.key$ holds. This contradiction proves that $^{\forall}i \in \{1, \ldots, M-1\}$, $|m(v_1) \wedge m(v_{i+1})| \leq |m(v_1) \wedge m(v_i)|$, i.e., a sequence $(|m(v_1) \wedge m(v_i)|)_{i=1}^{M}$ is weakly decreasing.
2) If $v_j.key \in T_k$, then we can prove it in the same way as 1).

■

Lemma 5: Let $(u_i)_i$ be the node sequence on the routing path for a `searchOp` query whose target key is k issued at

the node $v_1(= v_{s_1})$. Then, the node sequence $(v_{s_i})_i$ is a subsequence of $(u_i)_i$.

Proof: $(v_{s_i}.key)_i$ is strictly increasing from Theorem 1, and $(u_i.key)_i$ is also strictly increasing. Thus, it is sufficient to show that $\{v_{s_i}\}_i \subset \{u_i\}_i$.

For each level l, let $W_{k,l}$ be:

$$W_{k,l} \coloneqq \left\{ w \in V \;\middle|\; \begin{array}{l} |m(v_1) \wedge m(w)| = l \wedge \\ v_1.key < w.key \leq k \wedge \\ {}^{\forall}w' \in V, \\ \left[|m(v_1) \wedge m(w')| \geq l+1 \Rightarrow \right. \\ \left. w'.key \leq k \Rightarrow w'.key < w.key \right] \end{array} \right\}. \tag{28}$$

From routing algorithm of `searchOp`, we have $\{u_i\}_i = (\bigsqcup_{l=0}^{\infty} W_{k,l}) \sqcup \{v_1\}$ where $\sqcup$ denotes disjoint union. It means that it is sufficient to show that: for each level l,

$$\{\, v_{s_i} \mid |m(v_1) \wedge m(v_{s_i})| = l \,\} \subset W_{k,l}. \tag{29}$$

Herein, fix any level $l \in \{0, 1, \ldots\}$. If there does not i satisfying $|m(v_1) \wedge m(v_{s_i})| = l$, (29) holds. Suppose there exists such a i, and let j be the minimum number of i satisfying it. From Lemma 4 and Theorem 1, the following proposition holds:

$$(29) \Leftrightarrow \begin{array}{l} \neg^{\exists} w \in V \text{ s.t.} \\ |m(v_1) \wedge m(w)| \geq l+1 \wedge v_{s_j}.key \leq w.key \leq k. \end{array} \tag{30}$$

Suppose, for proof by contradiction, that

$${}^{\exists} w \in V \text{ s.t. } |m(v_1) \wedge m(w)| \geq l+1 \wedge v_{s_j}.key \leq w.key \leq k. \tag{31}$$

Let ι be the maximum number of i satisfying $|m(v_1) \wedge m(v_i)| \geq l+1$, and let l' be $|m(v_1) \wedge m(v_\iota)|$.

1) If $v_{s_j-1} \in S_k$, then $\iota = s_j - 1 = s_{j-1}$, i.e., the node v_ι selects the right adjacent node v_{s_j} at level l as the next node. On the other hand, from the hypothesis (31), the node v_ι should select the right adjacent node $(\neq v_{s_j})$ at level l' as the next node. This contradiction proves (29).
2) If $v_{s_j-1} \in T_k$:
   a) If $\iota = s_j - 1$, then the node v_ι has the left adjacent node v_{s_j} at level l and the left adjacent node w at level $l'(> l)$ where $v_{s_j}.key < w.key < k$. It contradicts a property of the topology of Skip Graph. Thus, (29) holds.
   b) If $\iota < s_j - 1$, the node v_ι selects the left adjacent node $v_{\iota+1}$ at level l as the next node where $v_{\iota+1}.key \in T_k$. In addition, v_ι has the left adjacent node w at level l' where $v_{s_j}.key \leq w.key \leq k$. From Lemma 3, we have $v_{s_j}.key \succ_k v_{\iota+1}$. From Lemma 1, we have $w.key \succ_k v_{\iota+1}$. Thus, the node v_ι should select the left adjacent node w at level l' as the next node instead of $v_{\iota+1}$. This contradiction proves (29).

Therefore, $(v_{s_i})_i$ is a subsequence of $(u_i)_i$. ■

Lemma 6: The length of $(v_{s_i})_i$ is expected $O(\log n)$.

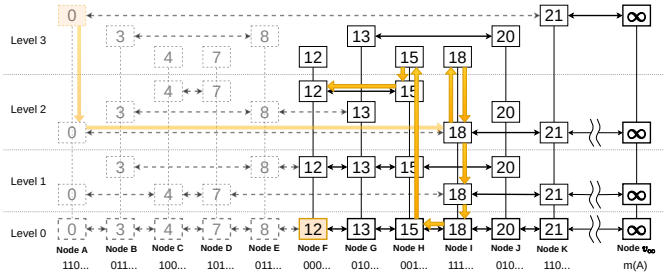

Fig. 7. A topology G' for the topology G, the start node $v_1 = A$, and the target key $k = 12$ of Fig. 6.

Proof: From Lemma 5, the lengths of $(v_{s_i})_i$ is expected $O(\log n)$ since the routing path length of `searchOp` is expected $O(\log n)$. ■

Lemma 7: Suppose there exists t_1, in other words, at most 1 detour route is used in the routing of `searchDSGOp`. Let $(u_i)_i$ be the node sequence on the routing path for a `searchOp` query whose target key is k issued at the node v_{t_1}. Then, the node sequence $(v_{t_i})_i$ is a subsequence of $(u_i)_i$.

Proof: Consider a search query whose target key is k is being issued at the node v_{t_1}. Then, we can prove it in the same way as Lemma 5. ■

We would like to show that the length of $(v_{t_i})_i$ is expected $O(\log n)$. However, we cannot prove it in the same way as Lemma 6 because a node v_{t_1} is determined by the start node v_1 and the topology dependent on random numbers. Herein, let G be the current topology and we define a topology G' for G, v_1, and k as follows:

- Let $K' \coloneqq K \cup \{\infty\}$ where ${}^{\forall}x \in K, x < \infty$.
- Let v'_∞ be a node with $v'_\infty.key = \infty$ and $m(v'_\infty) = m(v_1)$.
- Let $V' \coloneqq \{\, v \in V \mid v.key \in T_k \cup \{k\} \,\} \cup \{v'_\infty\}$.
- Let G' be the Skip Graph topology determined by V'.

For example, if G, v_1, and k are the topology, the start node, and the target key in Fig. 6, respectively; the topology of Fig. 7 shows G'. Then, the following lemmas hold.

Lemma 8: Let $(u'_i)_i$ be the node sequence on the routing path for a `searchOp` query whose target key is k issued at the node v'_∞ in the topology G'. Then, the node sequence $(v_{t_i})_i$ is a subsequence of $(u'_i)_i$.

Proof: If there does not exist t_1, then $(v_{t_i})_i$ is a subsequence of $(u'_i)_i$ since $(v_{t_i})_i$ is empty.

Suppose there exists t_1. Let $(u_i)_i$ be the node sequence on the routing path for a `searchOp` query whose target key is k issued at the node v_{t_1} in the topology G. From Lemma 7, $(v_{t_i})_i$ is a subsequence of $(u_i)_i$. Thus, it is sufficient to show that $(u'_i)_i$ contains a node v_{t_1}, because it means that $(u_i)_i$ equals the contiguous subsequence of $(u'_i)_i$ where the first element is v_{t_1} and the last element is the last node of $(u'_i)_i$.

Let v'_{max} be the node with the maximum key in $\arg\max_{v' \in V' - \{v'_\infty\}} \{\, |m(v_1) \wedge m(v')| \,\}$ (in Fig. 7, v'_{max} denotes a node K). From the definition, v'_{max} equals a node u'_2.

1) If $|m(v_1) \wedge m(v_{t_1})| = |m(v_1) \wedge m(v'_{max})|$, then $v_{t_1}.key \leq v'_{max}.key$ holds from a property of the

topology G. Thus from routing algorithm of `searchOp`, $(u'_i)_i$ contains v_{t_1}.

2) If $|m(v_1) \wedge m(v_{t_1})| < |m(v_1) \wedge m(v'_{max})|$, for each v_i $(i = 1, \ldots, t_1 - 2)$, there does not exist a right adjacent node w of v_i where $w.key \in T_k \wedge |m(v_1) \wedge m(w)| > |m(v_1) \wedge m(v_{t_1})| \wedge v_{i+1}.key \succ_k w.key$. From Lemma 2, Lemma 3, and a property of the topology G, it means that there does not exist a node $w \in V$ where $|m(v_1) \wedge m(w)| > |m(v_1) \wedge m(v_{t_1})| \wedge k \leq w.key \leq v_{t_1}.key$. Thus from routing algorithm of `searchOp`, $(u'_i)_i$ contains v_{t_1}.

Therefore, $(v_{t_i})_i$ is a subsequence of $(u'_i)_i$. ■

Lemma 9: The length of $(v_{t_i})_i$ is expected $O(\log n)$.

Proof: From Lemma 8, it is sufficient to show that the length of $(u'_i)_i$ is expected $O(\log n)$.

Let $\{U'_w\}_{w \in \Sigma^*}$ be the family of doubly linked lists of the topology G'. Each U'_w denotes the set of nodes $v \in V$ with $m(v) \uparrow |w| = w$. Then, the sequence $U'_{m(v_\infty)\uparrow 0}, U'_{m(v_\infty)\uparrow 1}, U'_{m(v_\infty)\uparrow 2}, \ldots$ is the skip list restriction of the node v_∞. Hence, we can show the length of $(u'_i)_i$ is expected $O(\log n)$ in the same way as the proof for message complexity of `searchOp` given in [7]. ■

Theorem 3: The routing path length M of `searchDSGOp` is expected $O(\log n)$.

Proof: From definitions of $(v_{s_i})_i$ and $(v_{t_i})_i$,

$$M \leq \left[\text{ the length of } (v_{s_i})_i \right] + \left[\text{ the length of } (v_{t_i})_i \right] + 1. \tag{32}$$

From Lemma 6 and Lemma 9, the lengths of $(v_{s_i})_i$ and $(v_{t_i})_i$ are expected $O(\log n)$, respectively. Therefore, M is also expected $O(\log n)$. ■

IV. Evaluation

We evaluated the path lengths by conducting routing simulations and observed the effect of the proposed method. The network topologies for the experiments are constructed as Skip Graph topologies with the following key generation methods:

- Generated by uniform distribution.
- Generated by power-law distribution.
- Random English titles on Wikipedia.
- Hashed Random English titles on Wikipedia.

It is important to evaluate path lengths for various key distributions because the routing paths depend on not only the topology, but also the key distribution for participating nodes.

Moreover, the routing algorithms used as the evaluation subjects are `searchOp`, `searchDROp`, `searchMLOp`, `searchDSGOp`, and `searchNLIOp`. Table I is a correspondence table between the names of the algorithms and the section numbers with their descriptions.

Herein, we introduce a routing algorithm `searchNLIOp`, which is "ideal" for using detour routes. In Detouring Skip Graph, mid is set as a function before the construction of the topology to estimate the key of the center of the lower-level node sequence. However, each node can perfectly determine the use of detour routes without the estimation if the node sequence information is known. We define `searchNLIOp` as a routing algorithm performing such perfect detour judgment and traversing from maximum levels. Although this routing is not practical due to the additional large cost of obtaining the node sequence information, we use it as a comparison target in the experiments to show the limitations of the proposed method.

TABLE I
Routing algorithms used as the evaluation subjects.

`searchOp`	: Skip Graph	(Sec. II-A)
`searchDROp`	: Utilizing detour routes	(Sec. III-A)
`searchMLOp`	: Traversing from max level	(Sec. III-B)
`searchDSGOp`	: Detouring Skip Graph	(Sec. III-C)
`searchNLIOp`	: Using node list information	(Sec. IV)

A. Generated by Uniform Distribution

We used pseudorandom numbers to generate keys so that the keys of participating nodes follow uniform distribution $P\{v.key = k\} = \frac{1}{2^{30}}$ $(k \in \{0, 1, \ldots, 2^{30} - 1\})$ where $v.key$ is the key of a node v regarded as a random variable. Then, the center estimation mid for this distribution is

$$mid_{uniform}(k_1, k_2) := \frac{k_1 + k_2}{2}. \tag{33}$$

This evaluation gives the effectiveness of the proposed method on unbiased key distribution.

On the topology constructed based on the keys generated by the above method, every node issued 100 search queries whose target keys are the keys of randomly sampled nodes. We plotted the average and the maximum of all routing path lengths for these queries in Fig. 8 and Fig. 9, respectively. The horizontal axis represents the number of participating nodes in increments of 100, and the vertical axis represents the average and the maximum path length, respectively. Each line corresponds to each routing method, where `searchDSGOp`(`mid : uniform`) and `searchDROp`(`mid : uniform`) represent routing that involves the use of $mid_{uniform}$ as a center estimation for the keys. Further, every routing method is executed on the same topology for each number of nodes; and each topology is built by adding nodes to the existing topology, rather than rebuilt from scratch each time. These conditions are the same for the other experiments discussed in the subsequent sections.

Moreover, we plotted the path length distribution on the above experiments where the number of participating nodes is 10000 in Fig. 10. The horizontal axis represents path lengths, and the vertical axis represents the frequency of each path length.

As a result in Fig. 8, the average path lengths are shorter in the order of `searchNLIOp`, `searchDSGOp`(`mid : uniform`), `searchDROp`(`mid : uniform`), `searchMLOp`, and `searchOp`. Furthermore, `searchDSGOp`(`mid : uniform`) of Detouring Skip Graph shortens the average path lengths by about 30% compared to `searchOp` of Skip Graph, and `searchNLIOp` shortens them by about 32% compared to `searchOp`. It shows that Detouring Skip Graph takes advantage of detour routes to shorten the path lengths. Similar trends are shown in Fig. 9 and Fig. 10. The proposed method also provides more stable

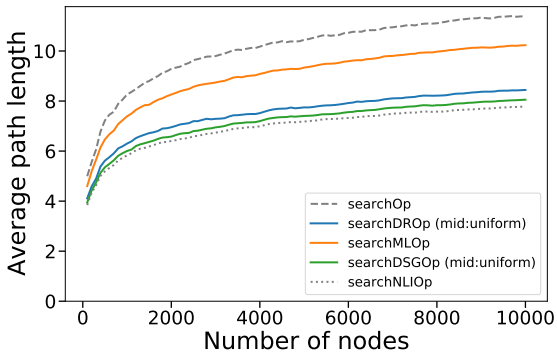

Fig. 8. Average path lengths on a topology whose keys were generated by uniform distribution. The target keys are the keys of randomly sampled nodes.

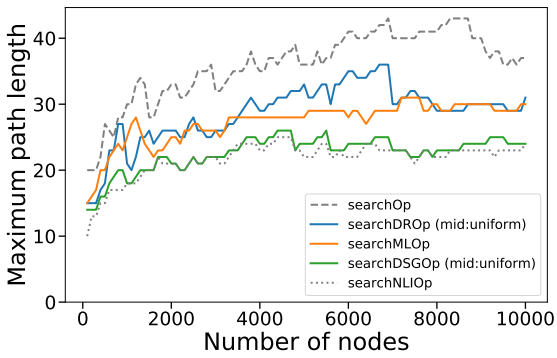

Fig. 9. Maximum path lengths on a topology whose keys were generated by uniform distribution. The target keys are the keys of randomly sampled nodes.

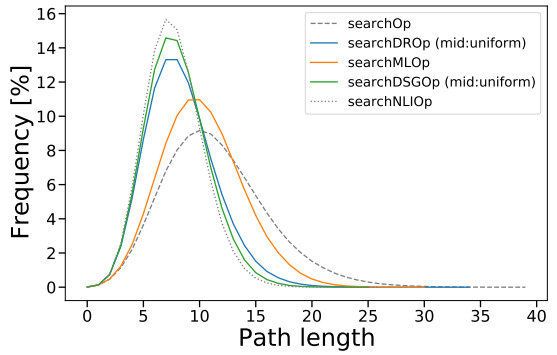

Fig. 10. Path length distribution on a topology whose keys were generated by uniform distribution where $n = 10000$. The target keys are the keys of randomly sampled nodes.

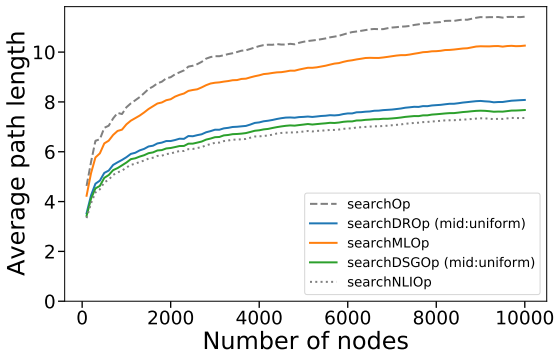

Fig. 11. Average path lengths on a topology whose keys were generated by uniform distribution. The target keys are generated by uniform distribution.

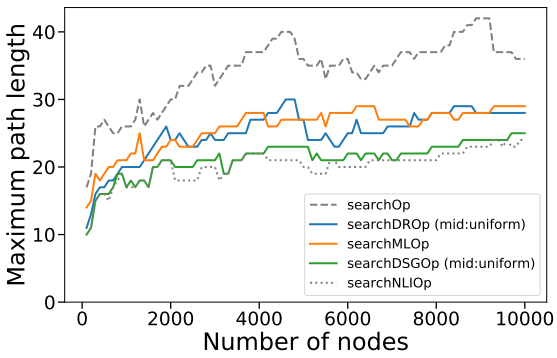

Fig. 12. Maximum path lengths on a topology whose keys were generated by uniform distribution. The target keys are generated by uniform distribution.

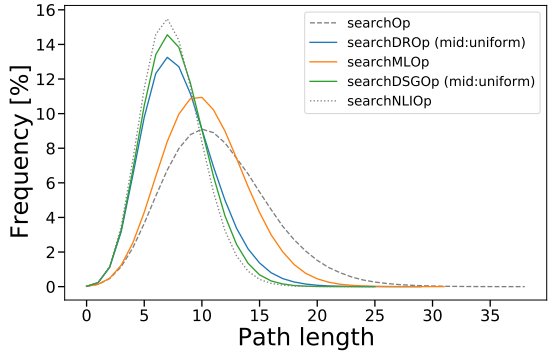

Fig. 13. Path length distribution on a topology whose keys were generated by uniform distribution where $n = 10000$. The target keys are generated by uniform distribution.

performance for any queries compared to Skip Graph since the standard deviations of the path lengths of `searchOp` and `searchDSGOp`(`mid` : `uniform`) are 4.59 and 2.78 in Fig. 10, respectively.

In addition, we conducted the same experiment on the condition that the target keys $k_{target} \in \{0, 1, \ldots, 2^{30} - 1\}$ are generated by uniform distribution for comparison purposes. In this evaluation, since the maximum number of nodes is 10^4, the probability that a target key is a non-existent key is $1 - \frac{10^4}{2^{30}} \simeq 1$. Thus, there does most likely not exist a target node for each target keys.

Fig. 11, Fig. 12, and Fig. 13 show the evaluation result. `searchDSGOp`(`mid` : `uniform`) of Detouring Skip Graph shortens the average path lengths by about 33% compared to `searchOp` of Skip Graph, and `searchNLIOp` shortens them by about 36% compared to `searchOp`. The result is similar to those for existent keys. We consider that it is because the distribution of target keys is the same in the two experiments.

B. Generated by Power-Law Distribution

We converted pseudorandom numbers to generate keys so that the keys of participating nodes follow power-law distribution $P\{v.key \leq k\} = \int_0^k f(\kappa)d\kappa$ $(0 \leq k \leq 2^{30})$ where $v.key$

TABLE II
AVERAGE PATH LENGTHS ON A TOPOLOGY WHOSE KEYS WERE GENERATED BY POWER-LAW DISTRIBUTION WHERE $n = 100, 1000, 10000$. THE TARGET KEYS ARE GENERATED BY UNIFORM DISTRIBUTION.

	$n = 100$	1000	10000
`searchOp`	4.87	8.17	11.50
`searchDROp(mid : uniform)`	4.10	6.30	8.47
`searchDROp(mid : power)`	4.09	6.27	8.45
`searchMLOp`	4.42	7.32	10.27
`searchDSGOp(mid : uniform)`	3.86	6.02	8.08
`searchDSGOp(mid : power)`	3.85	6.00	8.06
`searchNLIOp`	3.79	5.82	7.79

is the key of a node v regarded as a random variable, f denotes a probability density function $f(k) = ck^{10}$ $(0 \leq k \leq 2^{30})$, and c denotes a constant that satisfies $\int_0^{2^{30}} f(k)dk = 1$. Then, from (9), the center estimation mid for this distribution is

$$mid_{power}(k_1, k_2) := \left(\frac{k_1^{10+1} + k_2^{10+1}}{2}\right)^{\frac{1}{10+1}}. \quad (34)$$

The key distribution may be biased in the actual use of the proposed method without hashed keys. The purpose of using power-law distribution is to evaluate the effectiveness of the proposed method on simply biased key distribution.

On the topology constructed based on the keys generated by the above method, every node issued 100 search queries whose target keys are the keys of randomly sampled nodes. We plotted the average and the maximum of all routing path lengths for these queries in Fig. 14 and Fig. 15, respectively. `searchDSGOp(mid : power)` and `searchDROp(mid : power)` represent routing that involves the use of mid_{power} as a center estimation of keys. Moreover, we plotted the path length distribution where the number of participating nodes is 10000 in Fig. 16.

As a result in Fig. 14, the average path lengths were almost the same in `searchDSGOp(mid : uniform)` and `searchDSGOp(mid : power)`, and we discovered that using $mid_{uniform}$ as a detour criterion is effective even if the key distribution is biased. Table II lists the average path lengths where the number of nodes n is 100, 1000, and 10000. The numerical values also indicate that the average path lengths of the routing following `searchDSGOp(mid : uniform)` and `searchDSGOp(mid : power)` are almost the same. In both routing, the average path length of `searchOp` was shortened by about 30%. In addition, the result shows that Detouring Skip Graph takes advantage of detour routes to shorten the path lengths even in this experiments since `searchNLIOp` shortens the average path lengths by about 32%. Similar trends are shown in Fig. 15 and Fig. 16. The proposed method also provides more stable performance for any queries compared to Skip Graph since the standard deviations of the path lengths of `searchOp` and `searchDSGOp(mid : uniform)` are 4.54 and 2.76 in Fig. 16, respectively.

In addition, we conducted the same experiment on the condition that the target keys $k_{target} \in \{0, 1, \ldots, 2^{30} - 1\}$ are generated by uniform distribution for comparison purposes. In this evaluation, all of them are non-existent keys.

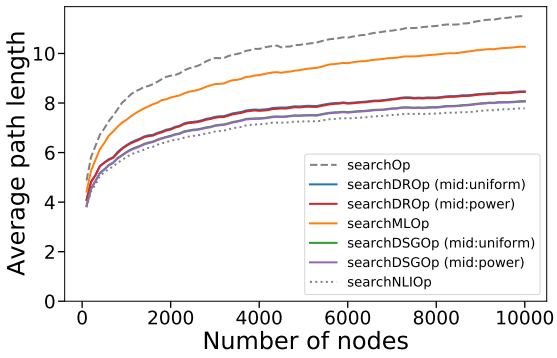

Fig. 14. Average path lengths on a topology whose keys were generated by power-law distribution. The target keys are the keys of randomly sampled nodes.

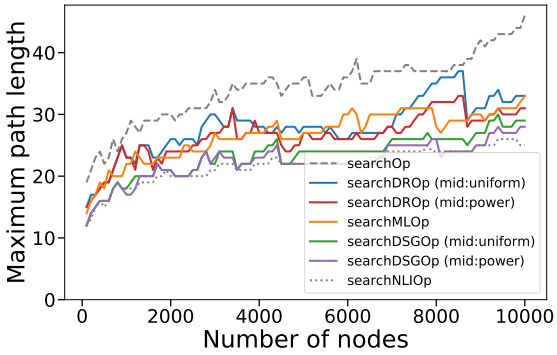

Fig. 15. Maximum path lengths on a topology whose keys were generated by power-law distribution. The target keys are the keys of randomly sampled nodes.

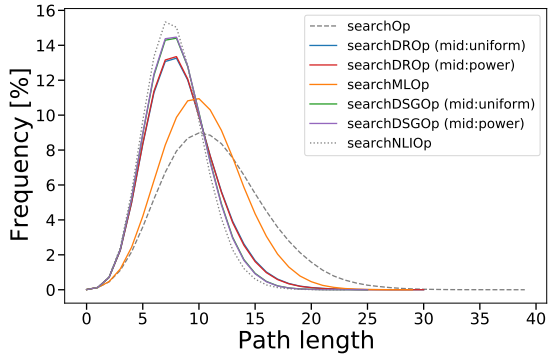

Fig. 16. Path length distribution on a topology whose keys were generated by power-law distribution where $n = 10000$. The target keys are the keys of randomly sampled nodes.

Fig. 17, Fig. 18, and Fig. 19 show the evaluation result. `searchDSGOp(mid : uniform)` and `searchDSGOp(mid : uniform)` of Detouring Skip Graph shortens the average path lengths by about 21% compared to `searchOp` of Skip Graph, and `searchNLIOp` shortens them by about 22% compared to `searchOp`. The shortening rate by the proposed method is smaller than that in the experiment with existent keys. We consider that it is because the target keys are biased relative to the key distribution of the nodes, thus a few detour routes were used.

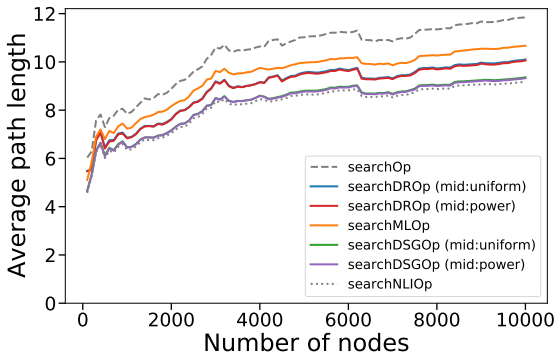

Fig. 17. Average path lengths on a topology whose keys were generated by power-law distribution. The target keys are generated by uniform distribution.

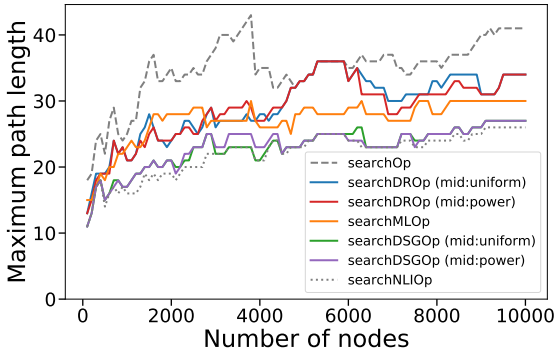

Fig. 18. Maximum path lengths on a topology whose keys were generated by power-law distribution. The target keys are generated by uniform distribution.

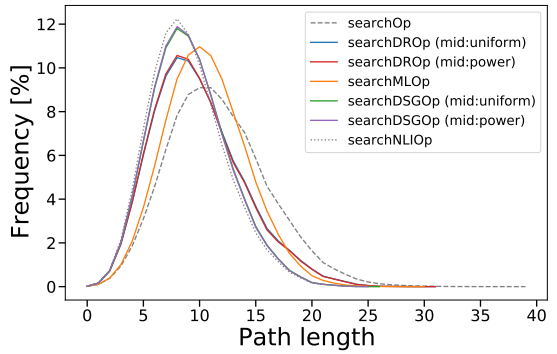

Fig. 19. Path length distribution on a topology whose keys were generated by power-law distribution where $n = 10000$. The target keys are generated by uniform distribution.

C. Random English Titles on Wikipedia

We used 10000 random English titles obtained on Apr. 15, 2020 from an API[2] published by Wikipedia as keys. Specifically, we encoded each title as a character string in UTF-8, and we used the strings as $256(= 2^8)$-based integer keys. The purpose of using random titles is to evaluate the effectiveness of the proposed method in realistic situations.

On the topology constructed based on the keys obtained by the above method, every node issued 100 search queries whose target keys are the keys of randomly sampled nodes. We plotted the average and the maximum of all routing path lengths for these queries in Fig. 20 and Fig. 21, respectively. Moreover, we plotted the path length distribution where the number of participating nodes is 10000 in Fig. 22.

As a result, `searchDSGOp(mid : uniform)` of Detouring Skip Graph shortens the average path lengths by about 26% compared to `searchOp` of Skip Graph, and `searchNLIOp` shortens them by about 31% compared to `searchOp`. The effectiveness of `searchDSGOp(mid : uniform)` is smaller than that of `searchNLIOp` because the detour criterion $mid_{uniform}$ is not a center estimation for this key distribution. However, the result shows that the proposed method has a large effect on shortening the path lengths despite the mismatch detour criterion. Similar trends are shown in Fig. 21 and Fig. 22. The proposed method also provides more stable performance for any queries compared to Skip Graph since the standard deviations of the path lengths of `searchOp` and `searchDSGOp(mid : uniform)` are 4.62 and 3.08 in Fig. 22, respectively.

D. Hashed Random English Titles on Wikipedia

Since Skip Graph is can be used as a DHT by hashing keys for load balancing, it is important to evaluate the effectiveness of the proposed method in such cases. In reality, the hash value of some properties of each node, e.g., the hashed IP address. In this scenario, we used the hash values of the titles of IV-C as keys using SHA3-512 hash function.

This experimental settings the same as IV-C except that the keys is hashed. Fig. 23, Fig. 24, and Fig. 25 show the evaluation result.

As a result, `searchDSGOp(mid : uniform)` of Detouring Skip Graph shortens the average path lengths by about 29% compared to `searchOp` of Skip Graph, and `searchNLIOp` shortens them by about 32% compared to `searchOp`. The proposed method also provides more stable performance for any queries compared to Skip Graph since the standard deviations of the path lengths of `searchOp` and `searchDSGOp(mid : uniform)` are 4.44 and 2.78 in Fig. 25, respectively.

Although the effectiveness is slightly less than that of IV-A, the result is similar. We consider that this is because the hash function brings the key distribution closer to uniform distribution and makes the detour criterion $mid_{uniform}$ appropriate. The result confirmed a sufficient effect for this experimental scale although the bias of the distribution is a concern with a small number of nodes. In addition, an important finding is that the proposed method is more effective with hashing than without it.

V. Conclusion

In this paper, we proposed Detouring Skip Graph, which shortens the path lengths by using effectively the topology that Skip Graph constructs. It introduces two techniques in the routing algorithm of Skip Graph: each node 1) utilizes detour routes and 2) traverses the adjacent nodes from the maximum level. The simple extension enables to apply the proposed method to existing Skip Graph. In addition, we proved the

[2]https://www.mediawiki.org/wiki/API:Main_page (accessed Apr. 15, 2020)

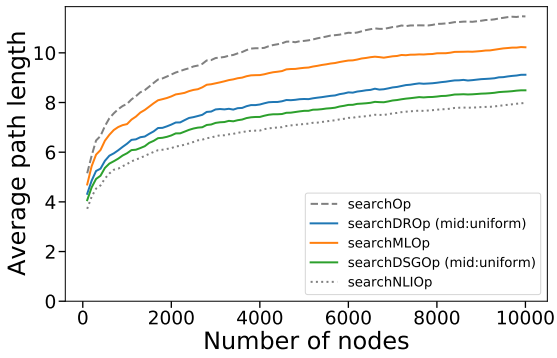

Fig. 20. Average path lengths on a topology whose keys are random English titles on Wikipedia.

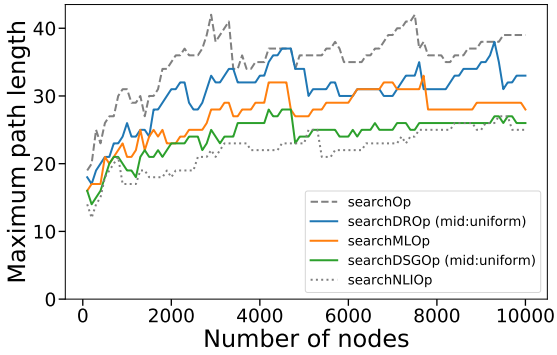

Fig. 21. Maximum path lengths on a topology whose keys are random Endlish titles on Wikipedia.

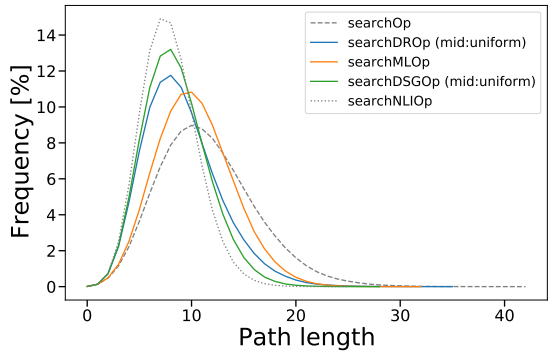

Fig. 22. Path length distribution on a topology whose keys are random English titles on Wikipedia where $n = 10000$.

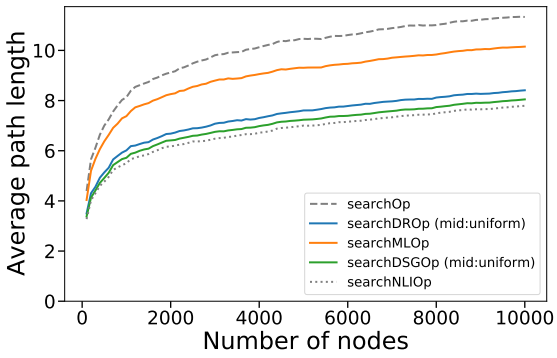

Fig. 23. Average path lengths on a topology whose keys are random English titles on Wikipedia.

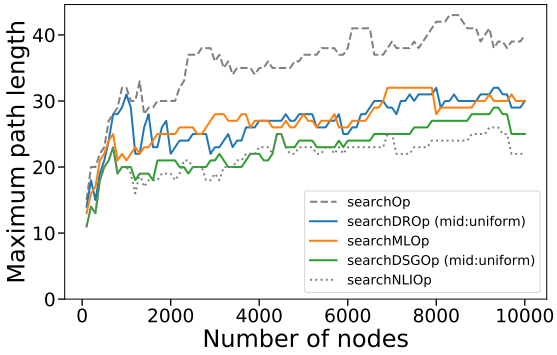

Fig. 24. Maximum path lengths on a topology whose keys are random Endlish titles on Wikipedia.

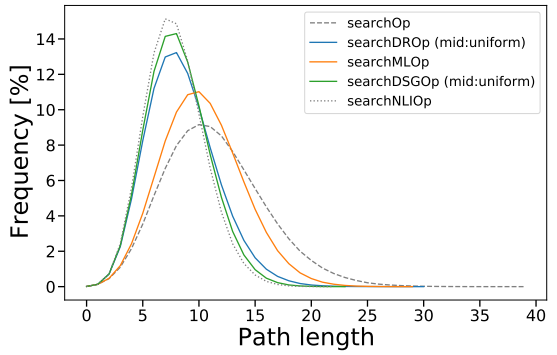

Fig. 25. Path length distribution on a topology whose keys are random English titles on Wikipedia where $n = 10000$.

reachability and correctness for any search query and that the routing path length is expected $O(\log n)$.

Detouring Skip Graph does not require construction of extra links and modification of its topology; thereby, it maintains the good properties of Skip Graph without additional costs. Through the evaluation experiments, we confirmed the large effect on shortening the path lengths, and especially the average path lengths were shortened by approximately 20%-30% in comparison with Skip Graph. Further, it was experimentally found that $mid_{uniform}$, the average of the keys belonging to two nodes, is effective as a detour criterion even for biased or realistic key distribution.

APPENDIX

In this appendix, we give Algorithm 4, which is a pseudocode of routing algorithm for search queries of Skip Graph.

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Algorithm 4: `searchOp` in node $v_{current}$

```
/* Skip Graph                                                    */
upon receiving ⟨searchOp, v_start, k_target, l_prev⟩ then
    if v_current.key = k_target then
        send ⟨foundOp, v_current⟩ to v_start ;
        return;
    else if v_current.key < k_target then
        for l_current ← l_prev downTo 0 do
            v_next ← v_current.neighbors[R][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≤ k_target then
                send ⟨searchOp, v_start, k_target, l_current⟩ to v_next;
                return;
    else
        for l_current ← l_prev downTo 0 do
            v_next ← v_current.neighbors[L][l_current];
            if v_next = ⊥ then
                continue;
            if v_next.key ≥ k_target then
                send ⟨searchOp, v_start, k_target, l_current⟩ to v_next;
                return;
    send ⟨notFoundOp, v_current⟩ to v_start
```


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Takeshi Kaneko received the B.S. degree in information science from Tokyo Institute of Technology, Tokyo, Japan, in 2019. He is currently a master student at Tokyo Institute of Technology. His research interests include distributed systems.

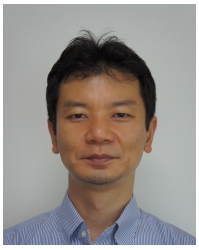
Ryohei Banno (M'17) received the Bachelor of Engineering degree and Master of Information Science and Technology degree from Hokkaido University, Sapporo, Japan, in 2010 and 2012 respectively, and the Ph.D. degree in science from Tokyo Institute of Technology, Tokyo, Japan, in 2018.

From 2012 to 2018, he was a Researcher in NTT Network Innovation Laboratories. From 2018 to 2020, he was a Researcher in Tokyo Institute of Technology. Since 2020, he has been an Assistant Professor in Kogakuin University, Tokyo, Japan. His research interests include distributed systems and Internet of Things (IoT).

Dr. Banno's awards and honors include the Outstanding Paper Award from Information Processing Society of Japan (IPSJ) in 2015, Inoue Research Award for Young Scientist from Inoue Foundation for Science in 2020, and Funai Research Award from Funai Foundation for Information Technology in 2020.

Kazuyuki Shudo (M'12) received the B.E. degree in 1996, the M.E. degree in 1998, and the Ph.D. degree in 2001 all in computer science from Waseda University. He worked as a Research Associate at the same university from 1998 to 2001. He later served as a Research Scientist at National Institute of Advanced Industrial Science and Technology. In 2006, he joined Utagoe Inc. as a Director, Chief Technology Officer. Since December 2008, he currently serves as an Associate Professor at Tokyo Institute of Technology. His research interests include distributed computing, programming language systems and information security.

Dr. Shudo has received the best paper award at SACSIS 2006, Information Processing Society Japan (IPSJ) best paper award in 2006, the Super Creator certification by Japanese Ministry of Economy Trade and Industry (METI) and Information Technology Promotion Agency (IPA) in 2007, IPSJ Yamashita SIG Research Award in 2008, Funai Prize for Science in 2010, The Young Scientists' Prize, The Commendation for Science and Technology by the Minister of Education, Culture, Sports, and Technology in 2012, and IPSJ Nagao Special Researcher Award in 2013. He is a member of IEEE, IEEE Computer Society, IEEE Communications Society and ACM.



Kota Abe (M'05) received his M.E. and Ph.D. degrees from Osaka University, Japan, in 1994 and 2000, respectively. In 1994, he joined Nippon Telegraph and Telephone Corporation (NTT), Japan. In 1996, he started working as a research associate at Media Center, Osaka City University, Japan. Since 2018, he has been a professor of Graduate School of Engineering, Osaka City University.

He received Information Processing Society of Japan (IPSJ) Best Paper Award in 2013. His research interests include distributed systems and system software.

Yuuichi Teranishi (M'09) received his M.E. and Ph.D. degrees from Osaka University, Japan, in 1995 and 2004, respectively. From 1995 to 2004, he was engaged Nippon Telegraph and Telephone Corporation (NTT). From 2005 to 2007, he was a lecturer of Cybermedia Center, Osaka University. From 2007 to 2011, he was an associate professor of Graduate School of Information Science and Technology, Osaka University. Since August 2011, He has been a research manager of National Institute of Information and Communications Technology (NICT).

He received IPSJ Best Paper Award in 2011. His research interests include technologies for distributed network systems and applications.